Wealth Inequality, Growth and Financial Bubbles

Mathieu Boullot

▶ To cite this version:

Mathieu Boullot. Wealth Inequality, Growth and Financial Bubbles. Economics and Finance. 2014. dumas-01105305

HAL Id: dumas-01105305

https://dumas.ccsd.cnrs.fr/dumas-01105305

Submitted on 20 Jan 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Université Paris1
UFR 02 Sciences Economiques
Master 2 Recherche ETE
Economie Theorique et Empirique
Année 2013-2014

Wealth Inequality, Growth and Financial Bubbles

Présenté et soutenu par : Mathieu Boullot
Directeur de la soutenance : Bertrand Wigniolle
Année de la soutenance : 2014

L'université de Paris 1 Panthéon Sorbonne n'entend donner aucune approbation, ni désapprobation aux opinions émises dans ce mémoire ; elles doivent être considérées comme propre à leur auteur.

1 Introduction

Some years from now a bubble busted on the US real estate market, with the consequences we know on the world economy. Until the collapse of Lehman Brothers in 2008, these consequences were largely underestimated. Six years after the crash, the US and European economies still did not fully recover from the huge financial and economic crisis that followed. The American economy is now recognized as having been driven since the 90's by two bubbles: the dot-com bubble which busted in the early 2000's and the real estate bubble. Similar phenomena were observed as well in Spain and Ireland. Japan has known and still knows a period of secular stagnation following such a boom-bust cycle on both stock and real estate markets in the 90's. Nowadays the probability of such events happening again in the forthcoming years or decades is not negligible. Thus understanding reccurent phenomena such as bubbles is of crucial importance for policy makers: could it be avoided ex-ante and if yes, how? How should fiscal and monetary authorities react ex-post?

This thesis is a first attempt at developping a common theory of inequality, financial bubbles and growth. We develop an OLG model with altruistic heterogeneous agents who differ only in their initial wealth. We include some non-convexities in the investment function, i.e a minimal level of investment, and some credit market imperfection, i.e a borrowing constraint. The combination of those features: altruism and initially unequal distribution of wealth, non-convexities in the invesment opportunities and a borrowing constraint, lead to the formation of a class society where the rich borrow from the poors in order to invest. The poors have no choice but to lend on the credit market. All the dynamic of our model goes through the credit market and the spread between the rate of return on capital and the rate of return on loans. If the former is higher than the latter, inequality are persistent in the long-run through the bequests and the initial distribution of wealth matters for the long-run distribution of wealth. Moreover we show that when agents have some special form of altruism, i.e family altruism, the consumption function is concave in wealth and the distribution of wealth has an impact on the capital accumulation. More unequal societies may be more likely to experiment capital over-accumulation at the competitive equilibrium. Furthermore we introduce financial bubbles in the model as pyramidal schemes and show that financial bubbles may exist in an economy which would have been otherwise dynamically efficient. Thus inequality in wealth may induce the appearance of financial bubbles in the economy through two new channels: first, inequality may foster the accumulation of capital and dynamic inefficiency; second, inequality may allow for financial bubbles even in a dynamically efficient economy and so as long as there is an interest rate spread.So, inequality may threaten the macroeconoic stability and redistributive policies may be necessary.

Related Literature

First, this thesis is related to the lieterature on endogeneous inequality and occupational choices under credit market imperfections. As in Aghon & Bolton (1997) and Piketty (1997), we introduce borrowing constraints and altruism to explain persistent and endogeneous inequality and show that the key is the credit market. On the contrary to Piketty, we show that a low interest rate on the credit market may not necessarly be bad for the poors who are constrained to make loans and can not invest. Through our model is way too simple to give clear conclusions on this topic, it seems that a low interest rate on the credit market favors the middle-class (potential investors) but hurts the poors (lenders). Our analysis however is much simpler than theirs and as Matsuyama (1998) we do not rely on any principal-agent problem between borrowers and lenders, but rather assume that agents may make default on their debt. As in Matsuyama (1998), inequality leads to the formation of two distinct classes in the society: the workers-lenders and the investors-borrowers.

This thesis is also related on the literature on the savings behaviors of the households. It seems to be that savings increase with wealth, which is a puzzle. Blinder (1975) provides a theory of bequests as luxury goods. Carroll & Kimball (1991) prove that the consumption function of the households may be concave if they face uncertainity due to precautionary savings motive. Jappelli & Pagano (1994) and Carroll & Kimball (2006) show that precautionary savings may also emerge due to the existence of borrowing constraints. And Chatterjee (1994) studies the macroeconomic implications of such non-linear behaviors in general equilibrium model. Based on this literature, we introduce family altruism as in Michel et al (2006) into the model and show that it generates such non-linearities and how it interacts with an inequalities.

As Tirole (1985) and Samuelson (1958), we introduce worthless assets in an OLG economy and prove that it can be traded at a positive price: bubbles. As in Martin & Ventura (2012), we explain how credit markets imperfections may lead to the emergence of financial bubbles. As in Cahuc & Challe (2009), our model features some form of occupational choice in a context of financial bubbles. As Mino (2007), we demonstrate the importance of intergenerational effects on the formation of financial bubbles.

2 The model

2.1 Households

Consider an economy inhabited by overlapping generations of young and old. There is no demographic growth, each generation consists of a continuum of agents of size normalized to 1. Time is discrete, it starts at time $t-1$ and then goes on forever. Individuals are assumed to live for two periods only. Being young they supply inelastically one unit of labor, get a wage income w_t and may receive a bequest b_t from their ascendants, the sum of the two constituting their total wealth ω_t. At the end of the period the young households decide how much to consume when young c_t^1, how much to consume when old c_{t+1}^2 and how much to bequeath to their descendant b_{t+1}. The bequests are made by the old households to the young households at the end of the period, before the youngs make their consumption and investment decisions.

Since individuals are risk-neutral, they will choose a portfolio that maximizes the expected return on their savings $\omega_t - c_t^1$. There are three means of savings in the economy: investment in capital i_t, loans on the credit markets s_t and a worthless asset x_t we will introduce in part two of this paper. Both the capital and credit markets are imperfect, each in a specific way.

On the credit market we assume that the households may default on their debt. Furthermore only a fraction $\lambda \in [0;1]$ of the expected returns on investment in capital are pledgeable [1]. Thus the equilibrium contract between lenders and borrowers on the credit market induces the existence of a borrowing constraint which ensures that the borrowers never decide to make default. Abstract from this borrowing constraint, the credit market is perfectly competitive and households may borrow from or lend to each other at a gross interest rate of ρ_{t+1} which will be endogeneously determined [2].

On the capital market we assume that there exist some indivisibilities. Individuals who invest less than a minimum level κ_0 will get a return on investment equal to 0. It is a common assumption in the literature on inequality and can be think of as an occupational or a technology choice: building capital requires some minimum scale of investment or it is worthless. The capital will be rented by the households to the firms at a rate R_{t+1} which will depend on the capital-labor ratio of the economy. Abstract

[1] λ is a measure of the imperfection of the financial markets. A perfect credit market would be associated to $\lambda \to \infty$.

[2] Note that ρ_{t+1} is the interest rate that prevails and will be determined at t on the credit market, but households' will have to repay their debt back at $t+1$.

from this indivisibility, the capital market is perfectly competitive.

The program of a young agent born at time t is the following:

$$\underset{\{c_t^1, c_{t+1}^2,\ b_{t+1}, s_t, i_t\}}{\text{Max}} \quad U(c_t^1, c_{t+1}^2, b_{t+1}) = u(c_t^1) + \beta \cdot u(c_{t+1}^2) + \gamma \cdot u(b_{t+1} + \theta \cdot w_{t+1}^e)$$

s.t.

$$\omega_t = c_t^1 + s_t + i_t \tag{1a}$$

$$\rho_{t+1} \cdot s_t + r_{t+1}^e \cdot i_t = c_{t+1}^2 + b_{t+1} \tag{1b}$$

$$\rho_{t+1} \cdot s_t + \lambda \cdot r_{t+1}^e \cdot i_t \geq 0 \tag{1c}$$

$$b_{t+1} \geq 0 \tag{1d}$$

$$c_t^1, c_{t+1}^2 > 0 \tag{1e}$$

$$r_{t+1}^e = \begin{cases} \delta_j \cdot R_{t+1}^e & \text{if} \quad i_t > \kappa_j \\ 0 \quad \text{otherwise} \end{cases} \tag{1f}$$

where $\forall j \in \{0...n\}, \quad \delta_j, \kappa_j \in \mathbf{R}_{++}, \quad \delta_j > \delta_{j-1}, \kappa_j > \kappa_{j-1}$ and $\delta_n = 1$

$X_{t+1}^e = \mathbb{E}_t X_{t+1}, \quad \theta \in [\ -1, 1\ ], \quad \beta, \gamma \in]\ 0, 1\ ], \quad \bar{c} \in [0; 1)$

(1a) and (1b) are the budget constraints when respectively young and old; (1c) is the borrowing constraint faced by the young agents; (1d) is a non-negativity constraint on bequests; (1f) is the technology constraint. For now we will assume that there exists only one technology, the $n-th$ which delivers a return of R_{t+1} for each unit invested if $i_t > \kappa$.

The utility function we use in this paper is a usual log-linear, separable and sub-additive one but with one modifications: as in Michel et al (2006), we include the expected wage of the children in the utility derived from bequests. But on the contrary to Michel et al (2006), we do not require θ to be positive and equal to 1. Both a positive or a negative θ may make sense: if $\theta > 0$ the parents care about the wealth of their children, family altruism; a $\theta < 0$ implies that the parents feel the need always to make a minimum level of bequest.

Both our special assumptions will induce some non-linearities in households' behaviors and we will study them separatly. Obviously a minimum consumption level puts the emphasis on intragenerational savings and can be think of as a tractable way to introduce precautionary savings (see Carroll & Kimball (1991, 2006)), whereas family altruism or a minimum bequest level puts the emphasis on the intergenerational channel and can be think of as a tractable way to introduce bequests as a luxury good (see Blinder (1975)).

The Kuhn-Tucker conditions for the maximization program are:

$$u'(c_t^1) - \Lambda_t - \mu_t = 0 \tag{2a}$$

$$\beta \cdot u'(c_{t+1}^2) - \frac{\Lambda_t}{\rho_{t+1}} = 0 \tag{2b}$$

$$\gamma \cdot u'(b_{t+1} + \theta \cdot w_{t+1}^e) - \frac{\Lambda_t}{\rho_{t+1}} - \xi_t = 0 \tag{2c}$$

$$\Lambda_t \cdot \{\frac{R_{t+1}^e}{\rho_{t+1}} - 1\} + \mu_t \cdot \{\lambda \cdot \frac{R_{t+1}^e}{\rho_{t+1}} - 1\} = 0 \tag{2d}$$

$$\Lambda_t \cdot \{\omega_t + i_t \cdot \{\frac{R_{t+1}^e}{\rho_{t+1}} - 1\} - c_t^1 - \frac{c_{t+1}^2}{\rho_{t+1}} - \frac{b_{t+1}}{\rho_{t+1}}\} = 0 \tag{2e}$$

$$\mu_t \cdot \{\omega_t^1 - c_t^1 + i_t \cdot \{\lambda \frac{R_{t+1}^e}{\rho_{t+1}} - 1)\} = 0 \tag{2f}$$

$$\xi_t \cdot b_{t+1} = 0 \tag{2g}$$

$$\xi_t, \Lambda_t, \mu_t \geq 0 \tag{2h}$$

$$\omega_t - s_t - i_t - c_t^1 = 0 \tag{2i}$$

Note that for now individuals have only two means of savings: invest on the capital market and making loans on the credit market. We shall introduce the worthless asset later. As can be seen from the Kuhn-Tucker conditions, we need to distinguish two cases: a binding financial constraint when the expected rate of return on capital investment R_{t+1}^e exceeds the rate of return on loans ρ_{t+1} and a non-binding financial constraint when both rates are equal. As we will explain later, the case $R_{t+1}^e < \rho_{t+1}$ will never be an equilibrium.

When $R_{t+1} = \rho_{t+1}$ the households are indifferent between making loans or investing in capital: the financial constraint never binds at the equilibrium.

$$c_t^1 = \frac{1}{A} \cdot \omega_t + \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \tag{3a}$$

$$c_{t+1}^2 = \frac{\beta}{A} \cdot \rho_{t+1} \cdot \omega_t + \frac{\beta}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \tag{3b}$$

$$b_{t+1} = \max\{\ 0, \frac{\gamma}{A} \cdot \rho_{t+1} \cdot \omega_t - \frac{1+\beta}{A} \cdot \theta \cdot w_{t+1}^e\ \} \tag{3c}$$

$$i_t\ \in [\ 0;\ \frac{A-1}{A} \cdot \varphi_t \cdot \omega_t - \frac{1}{A} \cdot \varphi_t \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\ [\tag{3d}$$

$$s_t\ \in] -\lambda \cdot \frac{R_{t+1}^e}{\rho_{t+1}} \cdot \{\frac{A-1}{A} \cdot \varphi_t \cdot \omega_t - \frac{1}{A} \cdot \varphi_t \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\};\ \frac{A-1}{A} \cdot \omega_t - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\ ] \tag{3e}$$

Where $A \equiv 1 + \beta + \gamma$ is the inverse of the marginal propensity to consume out of wealth and $\varphi_t \equiv \frac{1}{1-\lambda\cdot\frac{R^e_{t+1}}{\rho_{t+1}}} = \frac{i_t}{i_t-(-s_t)} \in [0;+\infty)$ is the degree of leverage associated to one unit of investment on the capital market: due to the financial market imperfection savings play a dual role as both a productive investment and a collateral. More savings allow for more debt and so more investment. The higher the spread between the return on capital and the borrowing cost, the higher this leverage effect: what matters for the lender is the solvability of the borrower and its willingness to repay back, which increases with the expected gains from the investment and decreases with the cost of the debt. Note that i_t is here total investment, that is investment financed both by the savings and by the debt.

If $R_{t+1} > \rho_{t+1}$ the households face an arbitrage opportunity: they can make money by borrowing at a low rate and invest this money at a higher return. Thus they will want to borrow and invest as much as possible: the borrowing constraint has to bind at the equilibrium.

$$c^1_t = \frac{1}{A}\cdot\omega_t + \frac{1}{A}\cdot\frac{1}{1+\phi_t}\cdot\theta\cdot\frac{w^e_{t+1}}{\rho_{t+1}} \tag{4a}$$

$$c^2_{t+1} = \frac{\beta\cdot(1+\phi_t)}{A}\cdot\rho_{t+1}\cdot\omega_t + \frac{\beta}{A}\cdot\theta\cdot w^e_{t+1} \tag{4b}$$

$$b_{t+1} = \max\{\ 0, \frac{\gamma\cdot(1+\phi_t)}{A}\cdot\rho_{t+1}\cdot\omega_t - \frac{1+\beta}{A}\cdot\theta\cdot w^e_{t+1}\ \} \tag{4c}$$

$$i_t = \frac{A-1}{A}\cdot\varphi_t\cdot\omega_t - \frac{1}{A}\cdot\frac{\varphi_t}{1+\phi_t}\cdot\theta\cdot\frac{w^e_{t+1}}{\rho_{t+1}} \tag{4d}$$

$$s_t = -\lambda\cdot\frac{R^e_{t+1}}{\rho_{t+1}}\cdot\{\frac{A-1}{A}\cdot\varphi_t\cdot\omega_t - \frac{1}{A}\cdot\frac{\varphi_t}{1+\phi_t}\cdot\theta\cdot\frac{w^e_{t+1}}{\rho_{t+1}}\} \tag{4e}$$

$\phi_t \equiv \frac{\frac{R^e_{t+1}}{\rho_{t+1}}-1}{1-\lambda\frac{R^e_{t+1}}{\rho_{t+1}}}$ is the gross return on each unit saved, i.e how much one more unit of saving allows to invest times the return on investment net of the borrowing costs. When ϕ_t is high the opportunity cost of first period consumption in terms of second period consumption is high: the households will save more and consume less during the first period. A higher interest rate spread implies a higher ϕ_t and thus lower first period consumption and higher savings.

2.2 Wealth inequality

If $R^e_{t+1} = \rho_{t+1}$, households are indifferent between being a worker-lender or an investor-borrower. But if $R^e_{t+1} > \rho_{t+1}$, every households would want to borrow and invest, thus there should be no supply of credit at all. To allow for to have two coexisting classes at the equilibrium even in the presence of an interest rate spread, we need to introduce one more assumption in our model: inequality in the initial distribution of wealth at $t-1$. In the presence of both imperfect credit markets and indivisibilities in the investment technology, individuals who are not wealthy enough to provide sufficiently high collaterals will not be able to borrow and then invest. As in our model the individuals are homogeneous in their abilities and as the labor market is assumed to be perfectly competitive, the only source of heterogeneity among households is inheritance.

Individuals who will receive large bequests will invest in capital and become investors, whereas the others will make loans to the former through the credit market. We shall refer to the first class of agents so as to the investors, unconstrained households or simply the rich, and to the second class so as to the constrained households, workers or simply the poors. As we shall demonstrate, the initial wealth distribution at time $t-1$ will be self-reproducing through the bequests decisions of the households: wealthier households bequeath more to their children who will be wealthy as well etc. Thus an agent whose ancestors were initially wealthy and investors will himself be wealthy and an investor.

More formally, let $G_{t-1}(\omega)$ be the initial cumulative distribution function of wealth among individuals at $t-1$ and $\Omega_{t-1} \equiv \int_0^1 \omega dG_{t-1}(\omega)$ be the initial aggregate wealth of the economy. To simplify the analysis we will restrain ourself to a case with two groups of homogeneous households, the "rich" (unconstrained) and the "poors" (constrained). We will assume that the first group represents a share ϵ of the population and each member owns initially (at time $t-1$) $\omega_{t-1} = \overline{\omega}_{t-1}$, whereas the second represents a share $(1-\epsilon)$ of the population and each members owns $\underline{\omega}_{t-1}$. Let $\underline{\Omega}_{t-1} \equiv \int_0^{1-\epsilon} \omega dG_{t-1}(\omega) = (1-\epsilon) \cdot \underline{\omega}_{t-1}$ and $\overline{\Omega}_{t-1} \equiv \Omega_{t-1} - \underline{\Omega}_{t-1} = \epsilon \cdot \overline{\omega}_{t-1}$ be the respective shares of the aggregate wealth owned by each group. Furthermore we will assume that $\overline{\omega}_{t-1}, \underline{\omega}_{t-1}, k_{t-1}$ and κ are such that the first group is always able to collect enough money to invest, whereas the second group never does or is indifferent between investing or not, but that both groups always bequeath.

In our model there will no social mobility at all: some dynastic families will always be investors whereas some others will always be workers. A consistent treatment of inequality would require a more realistic distribution of initial wealth, should model explicitly the occupational choice – notably define the inheritance threshold etc. – and so make ϵ endogeneous. Furthermore it should includes idiosyncratic shocks, heterogeneity in skills etc. Nevertheless our framework, through way too simple, is enough for the purpose of this study. Moreover and in comparison to for example Matsuyama (1998), the simplicity of our modeling to get deeper into the dynamic of the model and the various effects due to the interactions between imperfect financial markets, capital accumulation and inequality.

At the equilibrium the constrained households who have no choice but to lend money will make loans to the unconstrained, who will borrow and invest. The unconstrained households will adapt their investment and credit demand decisions depending on the interest rate spread. On the contrary the constrained households will always supply and consume the same amounts defined by (3), where $i_t = 0$ and $s_t = \omega_t - c_t^1$.

Proposition 1. *The average propensity to consume out of wealth is higher (lower) for the unconstrained households than for the constrained households if and only if $\theta > (<) 0$ Both are equal if and only if $\theta = 0$. The average propensity to bequeath is lower (higher) for the unconstrained households than for the constrained households if and only if $\theta > (<) 0$. If $\theta = 0$, bequests and consumption decisions are linear in wealth.*

Proof. The derivatives with respect to ω_t of $\frac{c_t^1}{\omega_t}$ and of $\frac{c_{t+1}^2}{\omega_t}$ are equal to 0 if and only if $\theta = 0$ and decrease (increase) with ω_t if and only if $\theta > (<) 0$. It is the reverse for $\frac{b_{t+1}}{\omega_t}$. The constrained households have a lower wealth than the unconstrained's by assumption. ■

Intuitivey the nature of the good "bequest" depends on the sign of θ: it is a normal good when $\theta = 0$, an inferior good when $\theta < 0$ and a superior good when $\theta > 0$. The implications of proposition (1) in a partial equilibrium point of view are straightforward: if $\theta > 0$, a more unequal distribution of wealth should imply a higher level of aggregate savings as aggregate average propensity to save increases with the concentration of wealth. This is so because the households who want to bequath more during second period need to save more during the first. Another implication is that a higer θ should imply a higher degree of correlation between one household's wealth and its parents': the children of the rich will receive higher bequests and be rich themselves. Naturally this is a ceteris paribus reasoning which as we shall see that this may not hold in a general equilibrium framework.

2.3 The productive sector

The firms have access to a cobb-douglas production function with constant return to scale in both capital and labor. They will rent capital from the investors and labor from all the households to produce the only good in the economy, which is used for both consumption and investment purposes. The good, labor and capital markets are perfectly competitive.

$$F(K_t; L_t) = K_t^{\alpha} L_t^{1-\alpha} = f(k_t) \tag{5}$$

Where we define $k_t \equiv \frac{K_t}{L_t}$ the capital stock per capita, which is also equal to the total capital stock as we normalize the total labor supply $L_t = 1$.

3 Bubbless equilibrium and the distribution of wealth

3.1 Short-run equilibrium

Definition 1 (Temporary Equilibrium). *Given the variables from previous period* $\{\omega_{t-1}(\omega), i_{t-1}(\omega)\}_{\omega\in[0;1]}$, *the expected wage rate* w^e_{t+1} *and the expected rate of return* R^e_{t+1}, *the temporay equilibrium of time* t *is defined by*

1. *the wage rate* w_t, *the rate of return* R_t *and the interest rate* ρ_{t+1},
2. *the aggregate variables* k_t, y_t,
3. *the individual variables* $c^1_t(\omega), c^2_t(\omega), b_t(\omega), i_t(\omega), s_t(\omega)$,
4. *the law of distribution of wealth* $G_t(\omega)$,

that satisfy the optimality conditions of the agents and the markets clearing conditions, where the agents are indexed by their wealth ω.

For now, there are only three markets: capital market, good market and credit market. We shall not consider the good market equilibrium in this study as we are mostly interested about the dynamic of wealth and capital accumulation, i.e the financial aspects of the economy. Thus we will focus on the other markets and invoke Walras' law to ensure that the good market clears at every period.

$$y_t(k_t) = \int_0^1 \{i_t(\omega) + c^1_t(\omega) + c^2_t(\omega)\} dG_t(\omega) \tag{6}$$

(6) is the market claring condition on the good market. Notice that bequests do not appear in equation (6) as they are transfers from consumption from one living agent to another. The idea is exactly the same with regard to the credit supply or credit demand.

Proposition 2. *The equilibrium interest rate on the credit market at time t, ρ_{t+1} satisfies*

- ρ_{t+1} *is unique and depends on the wealth distribution at* t*;*
- $\rho_{t+1} \in]\lambda \cdot R^e_{t+1};\ R^e_{t+1}]$*;*
- $\rho_{t+1} = \min\{\ \frac{\Omega_t}{\underline{\Omega}_t} \cdot \lambda;\ 1\} \cdot R^e_{t+1}$ *when* $\theta = 0$*;*
- *Given the values* $\underline{\Omega}_t$ *and* $\overline{\Omega}_t$*,* ρ_{t+1} *strictly increases with* λ *when* $\theta \geq 0$*;*
- *Given the values* $\underline{\Omega}_t$ *and* $\overline{\Omega}_t$*,* ρ_{t+1} *strictly increases with* θ*;*
- *Given the values* $\underline{\Omega}_t$ *and* $\overline{\Omega}_t$*,* ρ_{t+1} *striclty increases (decreases) with* ϵ *when* $\theta >$ $(<)0$ *and* $\lambda > \frac{1}{2}$*. It strictly decreases (increases) with* ϵ *when* $\lambda < \frac{1}{2}$ *and* $\theta > (<)0$.

Proof. If $\rho_{t+1} > R^e_{t+1}, I_t = \int_0^1 i_t(\omega)dG_t(\omega) = 0$, and if $\rho_{t+1} < \lambda \cdot R^e_{t+1}, I_t = \int_0^1 i_t(\omega)dG_t(\omega) = +\infty$, where I_t is aggregate investment demand. Thus $\rho_{t+1} \in]\lambda \cdot R^e_{t+1};\ R^e_{t+1}]$ [3]. For the rest of the proof, we need to write the credit market clearing condition

$$\int_0^{1-\epsilon} \{\frac{A-1}{A} \cdot \omega_t - \frac{1}{A} \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}}\} dG_t(\omega) \tag{7}$$

$$- \lambda \cdot \frac{R^e_{t+1}}{\rho_{t+1}} \cdot \int_\epsilon^1 \{\frac{A-1}{A} \cdot \varphi_t \cdot \omega_t - \frac{1}{A} \cdot \frac{\varphi_t}{1+\phi_t} \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}}\} dG_t(\omega) = 0$$

$$\Leftrightarrow (A-1) \cdot \{\lambda \cdot \frac{R^e_{t+1}}{\rho_{t+1}} \cdot \varphi_t \cdot \overline{\Omega}_t - \underline{\Omega}_t\} = -\theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \{(1-\epsilon) + \epsilon \cdot \frac{\lambda}{1-\lambda}\} \tag{8}$$

Multiplying both sides of (...) by ρ_{t+1} and rearranging, taking the derivatives and the limit, we show that ρ_{t+1} is uniquely determined. Both the RHS and LHS are obviously decreasing with ρ_{t+1}; the RHS is always of the sign of $-\theta$. Our other propositions are immediately derived from the above equation. ■

If $\theta = 0$, the LHS represents the excess demand for debt for investment purpose by the rich, which is obviously decreasing with ρ_{t+1}: as long as $\rho_{t+1} < R_{+1}$ the rich have an arbitrage opportunity, they want to borrow as much as possible and the borrowing constraint is binding. What matters for this effect is the total wealth owned by each class of agents. Very intuitively, the savings and investment functions of the households are linear to the wealth – abstract from this θ. Thus the wealthier the unconstrained households, the higher the demand for debt, and the higher the wealth of the constrained households, the higher the supply of debt. The equilibrium interest rate has to equate demand for debt and supply for debt: as $\Omega_t \equiv \underline{\Omega}_t + \overline{\Omega}_t$, the higher

[3] If $\rho_{t+1} < \lambda \cdot R^e_{t+1}$, the agents would like to borrow as much as possible to default on their debt.

the ratio of total wealth to wealth of the poors and the higher the excess demand for debt. Note that the interest rate is equal to the rate of return on capital if and only if $\lambda \cdot \Omega_t = \underline{\Omega}_t \Leftrightarrow \frac{\lambda}{(1-\lambda)} \cdot \overline{\Omega}_t = \underline{\Omega}_t$, that is if the pledgeable wealth of the rich times their leverage is equal to the wealth of the poors. It is quite natural that the interest rate increases with λ: the poors do not want to save more if λ increases, but the rich can borrow and want to borrow more.

The RHS represents the demand for savings due to the intergenerational effect, which is independant of the level of wealth. For $\theta > 0$ both the poors and the rich wants to reduce their savings in comparison to the case with $\theta = 0$. But the rich have better possibilities to smooth consumption and get higher returns on their savings: they have a lower discount factor and want to increase their savings (or decrease their demand for debt) from a smaller amount than the poors do. The higher ϵ the stronger this effect because the higher the number of rich relative to the number of poors: the interest has to go up to give the rich the equilibrium-compatible incentives.To think of the effect of a small λ, keep in mind that here the rich save for two reasons: first to smooth consumption, second to provide a collateral for the debt. When λ is too small, the second effect is not worth the cost and even if the rich still save more than the poors, they do not save enough to provide a sufficient amount of collateral. Thus the interest rate has to decrease to give them incentives to invest more, provide more collateral and finally borrow the exact amount the poors want to lend. When $\theta < 0$ all the effects are reversed: better-but-not-perfect-functionning credit markets in this case may hurt the poors.

The temporary equilibrium may be expressed as

$$\int_0^1 \{i_t(\omega) + c_t^1(\omega) + c_t^2(\omega)\} dG_t(\omega) = y_t \tag{9}$$

$$k_t = \int_0^1 i_{t-1}(\omega) dG_{t-1}(\omega) = \epsilon \cdot i_{t-1}(\overline{\omega}_{t-1}) \tag{10}$$

$$y_t = f(k_t) \tag{11}$$

$$w_t = f(k_t) - f'(k_t) \cdot k_t \tag{12}$$

$$R_t = f'(k_t) \tag{13}$$

$$b_t = b_t(\omega_{t-1}) \tag{14}$$

$$c_t^2 = c_t^2(\omega_{t-1}) \tag{15}$$

$$c_t^1 = c_t^1(w_t, b_t, w_{t+1}^e, R_{t+1}^e, \rho_{t+1}) \tag{16}$$

$$i_t = i_t(w_t, b_t, w_{t+1}^e, R_{t+1}^e, \rho_{t+1}) \tag{17}$$

$$s_t = w_t + b_t(\omega_{t-1}) - i_t(w_t, b_t, w_{t+1}^e, R_{t+1}^e, \rho_{t+1}) \tag{18}$$

plus the equilibrium condition on the credit market (...).

Let us denote aggregate equilibrium savings (AS) as $\Gamma_t \equiv \Omega_t - \int_0^1 c_t^1(\omega)dG_t(\omega)$. For now, $\Gamma_t = \int_0^1 i_t(\omega)dG_t(\omega) + \int_0^1 s_t(\omega)dG_t(\omega) = \frac{A-1}{A} \cdot \Omega_t - \theta \cdot \frac{1}{A} \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \cdot \{(1-\epsilon) + \epsilon \cdot \frac{1}{1+\phi_t}\}$.

By definition, in our model aggregate savings are equal to wealth minus consumption, or as well to asset holdings. Note that AS unambiguously decreases with ρ_{t+1} as long as $\theta > 0$: the discount factor for the intergenerational effect of both the poors and the rich increases (more for the latter than the former as the profitability of leverage decreases); AS unambiguously increases with aggregate wealth but does not depend on the repartition of the wealth; the effect of the number of investors ϵ is ambiguous as from one hand the rich have a higer discount factor and save more, but from the other hand higher ϵ implies a higher ρ_{t+1}.

Our model clearly captures some of the possible of the concentration of wealth on AS, but only through the number of investors relative to the number of creditors. This is obviously a drawback of our too simple modeling: with CRS production function, homogeneous tastes and nothing like transactions costs or externalities, the identity of the investors does not matter and the non-linearity in the savings behaviors is independant on the wealth. Assuming as in Martin & Ventura (2012) that there may exist different investors with different technologies co-existing in the economy would add a more realistic effect: identity of the investors would matter as well as the repartition of wealth, but through investment efficiency and not directly through AS itself. This is another possibility of how inequality when combined with imperfect financial markets could ease the conditions on the existence of bubbles [4].

Proposition 3. *The wealth distribution at t is perfectly correlated to wealth distribution at $t-1$.*

Proof. All the households will earn the same wage rate w_t, but the bequests they receive differ: some receive high bequests and may become investors, whereas others receive low bequests and remain workers. As the bequests are increasing with wealth, und furthermore the allocated share of wealth to bequests increases with wealth, the children of the constrained households will be constrained and the children of the unconstrained households will be unconstrained. ■

Remember that by assumption the minimal investment threshold κ, the initial capital

[4] For a review of the literature on inequality and investment opportunities/effiency, see Banerjee (2007).

stock k_t and the initial distribution of wealth $G_{t-1}(\omega)$ are such that households always make bequests but that the constrained households whether will always remain constrained or will be indifferent between investing or not.

Proposition 4. *The temporary equilibrium exists and is unique, conditionally to the state variables k_{t-1} and $G_{t-1}(\omega)$ and the expectations w^e_{t+1} and R^e_{t+1}.*

Proof. Given the capital stock k_t, $f(k_t)$, w_t and R_t are single-valued function. Given the wealth distribution of the previous period $G_{t-1}(\omega)$, the wealth distribution at t is uniquely determined. And given the expectations and the wealth distribution at t, the interest rate on the credit market ρ_{t+1} is uniquely determined. Then, all the decisions functions of the households are single-valued. ∎

Proposition 5. *If $\theta = 0$ and $\lambda > 0$ the aggregate variables at t are independant from the distribution of wealth at $t-1$ and can be expressed as a function of k_t only which is independant from the wealth distribution at $t-1$.*

Proof. By definition, $\Gamma_{t-1} = k_t$ or aggregate savings is equal to aggregate investment. And as we already proved, when $\theta = 0$ AS are independant from the wealth distribution. Or to do it in another way

$$\begin{aligned}
&\int_0^1 i_{t-1}(\omega) dG_{t-1}(\omega) = k_t \\
\Leftrightarrow\ & \frac{A-1}{A} \cdot \varphi_{t-1} \cdot \overline{\Omega}_{t-1} = k_t \\
\Leftrightarrow\ & \frac{A-1}{A} \cdot \Omega_{t-1} = k_t \qquad (19)
\end{aligned}$$

where we made use of the analytical expression of φ_t and of the equilibrium condition on the credit market when $\theta = 0$. The rest of the proof follows from the definition of the short-run equilibrium. ∎

(19) simply states that aggregate investment equals aggretate capital stock, where the aggregate investment does not depend on the wealth distrubiton. This may seem quite surprising at first view, but it is quite natural and intuitive: when savings behavior are linear with wealth, the distribution of wealth does not matter at all for the aggregates. It is actually the exact conclusion of the neo-classical growth model.

To explain the results of proposition (5) and the neutrality of the wealth distribution to AS, keep in mind that in our model everything goes through the credit market where agents who are heterogeneous in wealth interact with each other. The wealthy among the households would like to borrow and invest a lot when $R_{t+1} > \rho_{t+1}$, thus we could think that aggregate investment should increase. But there are two other effects: first, the rich want to increase their investment through the debt and not through a lower consumption, they will reduce their first period consumption if and only if it allows them to borrow more; second, the poorest households want to consume and save exactly the same amount, whatever the interest rate spread. So ρ_{t+1} has to increase until that condition (19) holds: the rich can not borrow more by increasing their savings, thus they do not and the wealth distribution is neutral for capital accumulation.

If $\lambda = 0$, there is no credit market at all and obviously the wealth distribution would matter: there may exist poverty traps and we would expect a more unequal society to accumulate more capital in the short-run.

Corollary 1. *If $\theta = 0$ and as long as $\lambda > 0$, there is no trade-off between equity and efficiency in the short-run: a more unequal distribution of wealth does not enhance growth nor threaten it.*

Proof. This is an immediate consequence of the neutrality of the distribution we found above. ■

3.2 Long-run equilibrium

Definition 2 (Inter-temporal Equilibrium)**.** *Given an initial wealth distribution $G_{t+1}(\omega)$ and an initial capital stock k_{t-1}, an inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibrium*

$$\{k_t, w_t, R_t, \rho_{t+1}, y_t, G_t(\omega), i_t(\omega), s_t(\omega), c_t^1(\omega), c_t^2(\omega), b_t(\omega)\}_{t=0}^{\infty}$$

that satisfies $\forall t \geq 0$ the following conditions

$$R_{t+1}^e = R_{t+1} \tag{20}$$

$$w_{t+1}^e = w_{t+1} \tag{21}$$

$$\int_0^1 i_t(\omega) dG_t(\omega) = k_{t+1} \tag{22}$$

$$\omega_t = w_t + b_t(\omega_{t-1}) \tag{23}$$

where (20) and (21) are the formation of expectations with perfect foresight, (22) and (23) are respectively the law of motion of the capital accumulation (equilibrium on the capital market) and the law of motion of wealth.

As there are only two groups of homogeneous households in our economy and we are interested by the evolution of wealth and capital accumulation, we may rewrite the inter-temporal equilibrium and consider the dynamic of the model only through the following system of differential equations

$$(A-1)\cdot\{\lambda\cdot\frac{R_{t+1}}{\rho_{t+1}}\cdot\varphi_t\cdot\overline{\Omega}_t-\underline{\Omega}_t\}+\theta\cdot\frac{w_{t+1}}{\rho_{t+1}}\cdot\{(1-\epsilon)+\epsilon\cdot\frac{\lambda}{1-\lambda}\}=0 \tag{24}$$

$$\overline{\Omega}_{t+1}-\epsilon\cdot(1-\frac{1+\beta}{A}\cdot\theta)\cdot w_t-\frac{\gamma\cdot(1+\phi_t)}{A}\cdot\rho_{t+1}\cdot\overline{\Omega}_t=0 \tag{25}$$

$$\underline{\Omega}_{t+1}-(1-\epsilon)\cdot(1-\frac{1+\beta}{A}\cdot\theta)\cdot w_t-\frac{\gamma}{A}\cdot\rho_{t+1}\cdot\overline{\Omega}_t=0 \tag{26}$$

$$k_{t+1}-\frac{A-1}{A}\cdot\varphi_t\cdot\overline{\Omega}_t+\frac{1}{A}\cdot\frac{\varphi_t}{1+\phi_t}\cdot\theta\cdot\frac{w_{t+1}}{\rho_{t+1}}=0 \tag{27}$$

Where (25) and (26) are the laws of motion of aggregate wealth of respectively investors and poors, (27) is the law of motion of capital accumulation and (24) is the equilibrium condition on the credit market. Replacing w_t and R_t by their expression as a function of k_t would give us a system of four differential equations with four unknowns: k, $\overline{\Omega}$, $\underline{\Omega}$ and ρ which determine all the dynamic of our model.

Proposition 6. *Given an intial level of capital stock k_{t-1} and an initial distribution of wealth $G_{t-1}(\omega)$, the equilibrium exists and is unique.*

Proof. See proposition (4) and note that the laws of motion of individuals wealth and capital accumulation are single-valued and that the production function satisfies the inada conditions. ■

Proposition 7. *When $\theta = 0$ and assuming that there is at least one unconstrained household, the aggregate variables always converge the same long-run level, independantly of the initial conditions on the wealth distribution.*

Proof. We already show that the capital accumulation at t is independant of the wealth distribution at $t-1$ and is uniquely determined by the aggregate wealth at $t-1$. Let us generalize this result. When $\theta = 0$, we can sum the individuals' wealth laws of motion and use the expression we found in proposition (2) for the equilibrium interest rate on the credit market to rewrite the system as

$$\frac{A-1}{A}\cdot\Omega_t=k_{t+1} \tag{28}$$

$$\Omega_t=(1-\alpha)\cdot k_t^{\alpha}+\frac{\gamma}{A}\cdot\alpha\cdot k_t^{\alpha-1}\cdot\Omega_{t-1} \tag{29}$$

which can be expressed as a function of k only and admits a unique fixed point $\tilde{k}$, with

$$\tilde{k} = \{\frac{A-1}{A} \cdot (1-\alpha) + \frac{\gamma}{A} \cdot \alpha\}^{\frac{1}{1-\alpha}} \tag{30}$$

$$\tilde{\Omega} = \Omega(\tilde{k}) = \frac{A}{A-1} \cdot \tilde{k} \tag{31}$$

■

When $\theta = 0$ and as we already explained, AS do not depend on the wealth distribution: we are in the neo-classical framework, thus the economy converges to a steady-state determined only by the technology and the tastes of the individuals. Note that the equilibrium level of capital will increase with the marginal propensity to save $\frac{A-1}{A}$ and with the labor-income share of output $1-\alpha$, decrease with the capital-share of output α and increase with the marginal propensity to bequeath $\frac{\gamma}{A}$. The more the individuals want to save during their lifetime and the more they will need to accumulate capital; the more the individuals want to bequeath, the higher the wealth of the youngs and thus the higher they will save; the effect of α is here not ambiguous as $A-1 > \gamma$ but it expresses that the bequests to the youngs which we could call intergenerational savings are based on capital-income whereas the lifetime savings are based on labor income. As the intergenerational marginal propensity to save is lower than the lifetime marginal propensity to save, capital accumulation has to decrease with α.

Note that the aggregate equilibrium wealth strictly increases with the aggregate equilibrium capital accumulation: the economy produces more and is so richer. Note as well that it has no real implications for the wealth of each individual: this aggregate wealth may be very unequally distributed as well shall demonstrate.

Corollary 2. *If $\theta = 0$ and as long as $\lambda > 0$, there is no trade-off between equity and efficiency in the long-run.*

Proposition (7) implies that when $\theta = 0$ and $\lambda > 0$, the distribution of wealth is neutral for the economy as a whole. It is so because we assumed homogeneous agents, competitive factor markets and a constant return to sale technology. Thus the identity of the investor does no matter for aggregate capital accumulation and wealth, neither the number of investors or how much each invests.

Proposition 8. *When $\theta = 0$, the aggregate equilibrium wealth owned by each group of agents and the interest rate spread will be conditional to the initial distribution of* *wealth $G_{t-1}(\omega)$, and more precisely on ϵ. Furthermore unequal steady-states will be associated to an interest rate spread, namely $\tilde{R} > \tilde{\rho}$.*

Proof. Let us write the equilibrium values of aggregate wealth of each class and the equilibrium condition on the credit market

$$\underline{\tilde{\Omega}} = (1-\epsilon) \cdot \tilde{w} \cdot \frac{1}{1 - \frac{\gamma}{A} \cdot \tilde{p}} \tag{32}$$

$$\tilde{\overline{\Omega}} = \epsilon \cdot \tilde{w} \cdot \frac{1}{1 - \frac{\gamma}{A} \cdot \frac{\tilde{R}(1-\lambda)}{1-\lambda \cdot \frac{\tilde{R}}{\tilde{\rho}}}} \tag{33}$$

$$\tilde{\rho} = \min\{\lambda \cdot \frac{\tilde{\Omega}}{\underline{\tilde{\Omega}}},\ 1\} \cdot \tilde{R} \tag{34}$$

Where $\tilde{R}, \tilde{w}$ and $\tilde{\Omega}$ are determined by the technology and tastes parameters, thus independantly of both ϵ and λ.
Depending on both ϵ and λ, we can check that some steady-state will be associated to an unequal distribution of wealth, i.e $\underline{\tilde{\Omega}} < \tilde{\overline{\Omega}}$ and $\frac{\underline{\tilde{\Omega}}}{1-\epsilon} < \frac{\tilde{\overline{\Omega}}}{\epsilon}$ whereas some others will be associated to an equal distribution of wealth at the aggregate level, i.e $\underline{\tilde{\Omega}} = \tilde{\overline{\Omega}}$ or at the individual level, i.e $\frac{\underline{\tilde{\Omega}}}{1-\epsilon} = \frac{\tilde{\overline{\Omega}}}{\epsilon}$, or both. A condition for an equal distribution of aggregate wealth at both levels is $\frac{\lambda}{1-\lambda} > \frac{1-\epsilon}{\epsilon}$ and $\epsilon = 0.5$. ∎

As Matsuyama (1998), we find that equal steady-state may arise if and only if the financial markets are not too imperfect, that is λ is not too low ($\lambda < 0.5$ would require $\epsilon > 0.5$ which does not make real sense). Furthermore the higher ϵ the more likely is the economy to converge to an equal steady-state. This is very intuitive as an equal steady-state requires that $\tilde{\rho} = \tilde{R}$ which will be favored by a high demand for credit and a low supply of credit. Ntoe that for ϵ small enough, it is possible to find steady-state with an equal distribution of wealth at the individual level but unequal at the macroeconomic level.

As we are here interested by the macroeconomic aspects of the distribution of wealth and its aggregate implications, we will not consider the wealth distribution at the individual level but rather speak of unequal steady-state when $\underline{\tilde{\Omega}} < \lambda \cdot \tilde{\Omega}$. Those steady-states are associated with an equilibrium spread which as we shall see could be crucial for the macroeconomic financial stability.

Proposition 9. *When $\theta > 0$ the equilibrium capital stock $\tilde{k}$*

- *is lower in comparison to the case when $\theta = 0$;*

- *decreases with α and A;*
- *increases with ϵ for λ small enough;*
- *decreases with λ for ϵ high enough;*

And the reverse for the equilibrium interest rate $\tilde{\rho}$. Furthermore, depending on the values of the parameters and on the initial distribution, the steady-state may or not be unequal.

Proof. See Appendix A. ■

The results of proposition (9) need to be misinterpreted: the fact that capital accumulation decreases when one include $\theta > 0$ is not a real surprise nor is the comparison between the two cases, $\theta = 0$ and $\theta > 0$ very interesting. The way we model non-linearity in wealth of the savings and consumption function is very tractable and allow us to get analytical results, but it is way too simple and unrealistic. As one can easily notice, this θ mechanically reduce the capital accumulation, whatever be the distribution of wealth. This θ shoud be only thought as a way to model in a simple way complex behaviors such as precautionnary savings and bequests as a luxury good, nothing more. Thus one shall better compare, θ being given, the level of capital at the equilibrium for different distribution of wealth. In this sense and in this sense only, the results of proposition (9) make sense and our study is interesting: under some assumptions on the capital markets, a more unequal distribution of wealth may not be neutral but rather promote capital accumulation. Thus more unequal wealth distributions may be more likely than more equal' to originate dynamic inefficiency as an equilibrium. And as we shall see now, this could be a threat for the stability of the economy as whole.

4 Bubbly equilibrium

4.1 The market for bubbles

Let us now assume the existence of another financial asset in the economy: bubbles. Following Tirole (1985), an asset will be bubbly if its market value is higher than its fundamental value, where the fundamental value is defiend as the discounted sum of the expected streams of dividends. Bubbles will be intrinsically worthless asset as it will be the case here, that is with asset a fundamental value of zero [5]. Bubbles are possible in an OLG economy because there are an infinite number of agents with finite lives. Those agents buy assets for two purposes: to invest and earn money, but as well as a way to save for the future. Since the famous work of Tirole (1985), it is well known

[5] See for example Samuelson (1958) where money is to be seen as a bubble.

that bubbles may appear in a dynamically inefficient OLG economy: agents are ready to buy a worthless asset if they expect to be able to resell it later at a higher rate, that is if they expect to realize capital gains. Intuitively this is possible if and only if the economy is dynamically inefficient: the returns on capital are negative and individuals invest too much in capital because they want to save a lot.

As in Martin & Ventura (2012), bubbles need to be think of as Ponzi-like pyramidal schemes: young agents buy the bubbles from the old agents and then themselves re-sell it when they are old. We will think there that some agents may be lucky enough to randomly create bubbles, that is to initiate such a pyramidal scheme, at no cost: it is a pure free lunch for those agents. This worthless asset we shall call bubbles will be traded in a new market, the bubbles market.

Let x_t be the total stock of existing bubbles at the beginning of the period t and x_t^N be the stock newly created bubbles during the period t. Agents who may create a bubble at the beginning of the period will sell it during the period (or keep it as a mean of savings). The capital gains from holding the bubbles from period t to period $t+1$ are the purely capital gains, such that the return on the bubbles is

$$q_{t+1}^e \equiv \frac{x_{t+1}^e}{x_t + x_t^N}$$
$$\text{where} \quad x_t^N = \int_0^1 x_t^N(\omega) dG_t(\omega)$$

The market for bubbles is perfectly competitive such that agents take the price as given and will buy the bubble if and only if it promises a return at least equal to the other assets (free disposability). We can compute the demand functions of the agents taking into account the new asset.

For the constrained agents:

$$\text{If} \quad q_{t+1} = \rho_{t+1}, \begin{cases} x_t \in [\ 0;\ \frac{A-1}{A} \cdot \varphi_t^q \cdot (\omega_t + x_t^N) - \frac{1}{A} \cdot \varphi_t^q \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\ [\\ s_t \in [\ -\lambda \cdot \frac{q_{t+1}^e}{\rho_{t+1}} \{\frac{A-1}{A} \cdot \varphi_t^q \cdot (\omega_t + x_t^N) - \frac{1}{A} \cdot \varphi_t^q \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\};\ \frac{A-1}{A} \cdot \omega_t - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}}\ [\\ i_t = 0 \end{cases}$$

Note that now the constrained agents may get indebted to buy the bubble, they do not rely anymore exlusively on the credit market to save money for the future.

$$\text{If} \quad q^e_{t+1} > \rho_{t+1}, \begin{cases} x_t = \frac{A-1}{A} \cdot \varphi^q_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \varphi^q_t \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \\ s_t = -\lambda \cdot \frac{q^e_{t+1}}{\rho_{t+1}} \{ \frac{A-1}{A} \cdot \varphi^q_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \varphi^q_t \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \} \\ i_t = 0 \end{cases}$$

Where $\varphi^q_t \equiv \frac{1}{1-\lambda \cdot \frac{q^e_{t+1}}{\rho_{t+1}}}$ is the leverage.
As it was the case for the unconstrained agents with investment in capital, if the returns on holding the bubbles are higher than the returns on the credit market, the constrained agents face an arbitrage opportunity. Thus they are willing to borrow as much as possible on the credit market to buy as much from the bubble as they can and the borrowing constraint has to bind.

If the investors is a unconstrained households

$$\text{If} \quad q^e_{t+1}, \rho_{t+1} < R^e_{t+1}, \begin{cases} x_t = 0 \\ s_t = -\lambda \cdot \frac{R^e_{t+1}}{\rho_{t+1}} \cdot \{ \frac{A-1}{A} \cdot \varphi_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \frac{\varphi_t}{1+\phi_t} \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \} \\ i_t = \frac{A-1}{A} \cdot \varphi_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \frac{\varphi_t}{1+\phi_t} \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \end{cases}$$

$$\text{If} \quad \rho_{t+1} < q^e_{t+1} = R^e_{t+1}, \begin{cases} x_t = \in [\ 0;\ \frac{A-1}{A} \cdot \varphi_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \varphi_t \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}}] \\ s_t = -\lambda \cdot \frac{q_{t+1}}{\rho_{t+1}} \{ \frac{A-1}{A} \cdot \varphi_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \varphi_t \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}} \} \\ i_t = \in [\ 0;\ \frac{A-1}{A} \cdot \varphi_t \cdot (\omega_t + x^N_t) - \frac{1}{A} \cdot \varphi_t \cdot \theta \cdot \frac{w^e_{t+1}}{\rho_{t+1}}] \end{cases}$$

If $\rho_{t+1} = q_{t+1} = R^e_{t+1}$, the borrowing constraint is not binding and the unconstrained households behave as we found earlier, except that now they are indifferent between making loans, investing in capital or buying the bubble. What is new now is that the constrained households will always reduce their credit supply and may even ask for debt to buy the bubble, if the return on the bubble is higher than the return on loans.

4.2 Short-run equilibrium

From now on, we will assume that $\theta = 0$. Having already proved that inequality may promote capital accumulation, letting $\theta \neq 0$ would just complicate the present analysis without bringing useful insights.

Definition 3 (Bubbly Temporary Equilibrium)**.** *A temporary equilibrium will be said* *to be bubbly if and only there exists a stock of bubbles $x_t > 0$ or newly created bubbles $x_t^N > 0$ such that the market for bubbles clear.*

Our definition of the temporary bubbly equilibrium is naturally more restrictive than the previous definition we offered for the temporary equilibrium.

Proposition 10. *The expected return on the bubble at the equilibrium has to satisfy*

- $q_{t+1}^e = \rho_{t+1} < R_{t+1}^e \Leftrightarrow \frac{x_t + \int_0^1 x_t^N(\omega) dG_t(\omega)}{\frac{A-1}{A} \cdot (\Omega_t + \int_0^{1-\epsilon} x_t^N(\omega) dG_t(\omega)) - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \cdot (1-\epsilon)} < 1$

- $q_{t+1}^e = \rho_{t+1} = R_{t+1}^e \Leftrightarrow \frac{x_t + \int_0^1 x_t^N(\omega) dG_t(\omega)}{\frac{A-1}{A} \cdot (\Omega_t + \int_0^{1-\epsilon} x_t^N(\omega) dG_t(\omega)) - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \cdot (1-\epsilon)} \geq 1$

And the bubble has to satisfy $0 \leq x_t \leq \Gamma_t = \Omega_t - \int_0^1 c_t^1(\omega) dG_t(\omega)$.

Proof. The case $q_{t+1} > R_{t+1}^e$ is ruled out by the Inada conditions on the production function: $\lim_{k \to 0} f'(k) = \infty$; the case $q_{t+1} < \rho_{t+1}, R_{t+1}^e$ makes no sense for us as there will exist no bubble.

By contradiction: if $R_{t+1}^e > q_{t+1} > \rho_{t+1}$ the borrowing constraint for both groups of households has to bind. Obviously this can not be an equilibrium. There are then two possibilities: whether $R_{t+1}^e = q_{t+1} = \rho_{t+1}$ or $R_{t+1}^e > q_{t+1} = \rho_{t+1}$.

If $\frac{x_t + \int_0^1 x_t^N(\omega) dG_t(\omega)}{\frac{A-1}{A} \cdot (\Omega_t + \int_0^{1-\epsilon} x_t^N(\omega) dG_t(\omega)) - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \cdot (1-\epsilon)} < 1$ the stock of bubble is strictly lower than the aggregate savings of the constrained households: they need to supply some funds on the credit market and the unconstrained households need to borrow and to invest in capital: the borrowing constraint has to bind. This requires $R_{t+1}^e > q_{t+1} = \rho_{t+1}$.

If $\frac{x_t + \int_0^1 x_t^N(\omega) dG_t(\omega)}{\frac{A-1}{A} \cdot (\Omega_t + \int_0^{1-\epsilon} x_t^N(\omega) dG_t(\omega)) - \frac{1}{A} \cdot \theta \cdot \frac{w_{t+1}^e}{\rho_{t+1}} \cdot (1-\epsilon)} \geq 1$ the stock of bubble is higher or equal to the aggregate savings of the poor: they do not supply any funds on the credit market. The unconstrained households need to buy the bubble (if the inequality is strict) and the borrowing constraint can not bind, thus it requires [6] $R_{t+1}^e = q_{t+1} = \rho_{t+1}$.

Obviously the bubble stock can not exceed the aggregate savings of the young agents, otherwise they could not buy it. ■

Intuitively, if the bubble is large enough (higher than the aggregate savings of the

[6] When we use the words "need", "require" etc. we have in mind the mechanism of the market which adjusts the price such that there is no excess demand nor excess supply at the equilibrium.

constrained households), the credit market just disappears. In this sense and as it was underlined by Martin & Ventura (2012), financial bubbles and credit markets are substitutes to each other in the presence of heterogeneous agents.

The market clearing condition on the credit market is now

$$\int_0^{1-\epsilon}\{\frac{A-1}{A}\cdot(\omega_t+x_t^N(\omega))-\frac{1}{A}\cdot\theta\cdot\frac{w_{t+1}^e}{\rho_{t+1}}-x_t(\omega+x_t^N(\omega))\}dG_t(\omega) \tag{35}$$
$$-\lambda\cdot\frac{R_{t+1}^e}{\rho_{t+1}}\cdot\int_\epsilon^1\{\frac{A-1}{A}\cdot\varphi_t\cdot(\omega_t+x_t^N(\omega))-\frac{1}{A}\cdot\frac{\varphi_t}{1+\phi_t}\cdot\theta\cdot\frac{w_{t+1}^e}{\rho_{t+1}}\}dG_t(\omega)=0$$

And the bubble market clearing condition is

$$x_t+\int_0^1 x_t^N(\omega)dG_t(\omega)-\int_0^1 x_t(\omega+x_t(\omega)^N)dG_t(\omega)=0 \tag{36}$$

Where we implicitly assume that the stock of bubbles is lower than the aggregate savings of the constrained agents, otherwise there owuld be no credit market at all.
As we see from (35), the effects of the bubbles will depend on the creator. If the creator is an unconstrained household, she will sell the bubble to the constrained households. Her wealth increases and with it her demand for debt. The effect on the interest rate on the credit market is unambiguous: it has to increase as both the supply of credit diminishes and the demand for it increases. The effects of bubbles creation by the constrained households seem at first view to be more complicated as a higher wealth increases the supply of credit, but a higher bubble stock reduces it. In fact, when one considers the saving function of the constrained households, the effects are very clear: bubbles, even if created by the constrained households, will necessarily increase the interest rate on the credit market. It is so because the unconstrained households have a marginal propensity to save lower than one: even if their wealth increases through the bubble creation, they will consume part of it and thus the supply of credit will be strictly reduced. If the total stock of bubbles is large enough, the credit market even disappears as unconstrained households do not supply any credit anymore.

Proposition 11. *Bubbles creation in the short-run always crowd-out capital accumulation.*

Proof. If the old bubble stock plus the newly created bubbles is larger than the savings of the constrained households, unconstrained households will buy the bubbles and it will obviously reduce their investment in capital. If new bubbles are created by constrained agents only such that the total bubble stock is lower than their aggregate savings, there will be a crowding out effect through the credit market

$$\begin{aligned} &\frac{A-1}{A}\cdot(\underline{\Omega}_t+\int_0^{1-\epsilon}x_t^N(\omega))-\frac{1}{A}\cdot\theta\cdot\frac{w_{t+1}^e}{\rho_{t+1}}-x_t(\omega+x_t^N(\omega))\}dG_t(\omega) \\ &-\lambda\cdot\frac{R_{t+1}^e}{\rho_{t+1}}\cdot\int_\epsilon^1\{\frac{A-1}{A}\cdot\varphi_t\cdot(\omega_t)-\frac{1}{A}\cdot\frac{\varphi_t}{1+\phi_t}\cdot\theta\cdot\frac{w_{t+1}^e}{\rho_{t+1}}\}dG_t(\omega)=0 \\ &x_t+\int_0^{1-\epsilon}x_t^N(\omega)dG_t(\omega)-\int_0^{1-\epsilon}x_t(\omega+x_t(\omega)^N)dG_t(\omega)=0 \end{aligned} \tag{37}$$

But as $x_t^N(\omega)=\frac{A-1}{A}\cdot(\underline{\Omega}_t+\int_0^{1-\epsilon}x_t^N(\omega))$, the interest rate on the credit market has to increase: the marginal propensity to save out of wealth is lower than one. Thus the agents who create a bubble will want to consume part of the supplement of wealth they get by selling the bubbles.Thus bubbles are non-neutral transfers, as some constrained agents will reduce their supply of credit one-to-one with the bubbles to buy it whereas some others will increase their supply of credit by a fraction smaller than one.
The idea is exactly the same if the wealthy create bubbles: they will sell it to the poors who will reduce their supply of credit. The unconstrained agents who were lucky enough to create a bubble will invest slightly more but consume more as well, whereas the other unconstrained agents will face a higher interest rate on the loan market: aggregate investment has to decrease. ■

Note also that aggregate wealth in the long-run will decrease due to the bubbles: it is quite natural as bubbles decrease capital accumulation but are not productive.

4.3 Long-run equilibrium

Definition 4 (Bubbly Inter-temporal Equilibrium)**.** *A bubbly inter-temporal equilibrium will be a sequence of temporary equilibrria*

$$\{k_t, w_t, R_t, \rho_{t+1}, y_t, G_t(\omega), i_t(\omega), s_t(\omega), c_t^1(\omega), c_t^2(\omega), b_t(\omega)\}_{t=0}^\infty$$

and a stochastic process

$$\{x_t, x_t^N(\omega)\}_{\omega\in[0,1],t\in[0,+\infty)}\}$$

such that for some $t\in[0;+\infty)$ the temporary equilibrium is bubbly.

The fundamental inter-temporal equilibrium, i.e non-bubbly, is characterized by

$$\{x_t, x_t^N(\omega)\}_{\omega\in[0,1],t\in[0,+\infty)}\}=\{0,0(\omega)\}_{\omega\in[0,1],t\in[0,+\infty)}$$

and is obviously a particular equilibrium among others. We will for now assume that there exists an initial stock of bubble in the economy and that agents do not create any bubbles.

Proposition 12. *When $\theta = 0$ a steady-state with an equal distribution of wealth may be bubbly if and only if the steady-state without bubble would have been dynamically inefficient.*

Proof. This is obviously the case studied by Tirole (1985). By contradiction: a steady-state with an equal distribution of wealth requires $\tilde{\rho} = \tilde{R}$, so one one need to have $\tilde{\rho}^b = \tilde{R}^b = \tilde{q}$ where b stands for bubbly. As bubbles reduce capital accumulation, $\tilde{R}^b > \tilde{R} > 1$ because the initial steady-state was dynamically efficient. But $x_{t+1} = q_{t+1} \cdot x_t$, thus one would have a stock of bubbles growing up to infinity. Which obviously contradicts the condition that the total stock of bubbles has always to be lower than Γ_t. ■

Proposition 13. *When $\theta = 0$ a steady-state with an unequal distribution of wealth may be bubbly even if the steady-state without bubbles would not have been dynamically inefficient if the bubble is lower than the aggregate savings of the constrained agents, $\lambda issmallenough$ and ϵ small enough. Furthermore the bubbly steady-state will be dynamically inefficient.*

Proof. Let us construct a bubbly steady-state with an unequal distribution of wealth. It requires $\tilde{R}^b > \tilde{\rho}^b = \tilde{q}^b = 1$ otherwise the stock of bubbles would grow up to infinity or asymptotically decrease to 0 in the long-run, and inequality at the steady state requires $\tilde{R}^b > \tilde{\rho}^b$. As bubbles are worthless assets, the long-run wealth of the agents is unchanged

$$\underline{\tilde{\Omega}} = (1 - \epsilon) \cdot \tilde{w} \cdot \frac{1}{1 - \frac{\gamma}{A}} \tag{38}$$

$$\overline{\tilde{\Omega}} = \epsilon \cdot \tilde{w} \cdot \frac{1}{1 - \frac{\gamma}{A} \cdot \frac{\tilde{R}(1-\lambda)}{1-\lambda \cdot \tilde{R}}} \tag{39}$$

The equilibrium condition on the credit, bubble and capital markets are

$$\tilde{x} - \frac{A-1}{A} \cdot \tilde{\underline{\Omega}}^b \leq 0 \tag{40}$$
$$\frac{A-1}{A} \cdot \tilde{\underline{\Omega}}^b - \lambda \cdot \tilde{R}^b \cdot \frac{A-1}{A} \cdot \tilde{\varphi}^b \cdot \tilde{\overline{\Omega}}^b \leq 0 \tag{41}$$
$$\frac{A-1}{A} \cdot \tilde{\underline{\Omega}}^b - \tilde{x} - \lambda \cdot \tilde{R}^b \cdot \frac{A-1}{A} \cdot \tilde{\varphi}^b \cdot \tilde{\overline{\Omega}}^b = 0 \tag{42}$$
$$\frac{A-1}{A} \cdot \tilde{\varphi}^b \cdot \tilde{\overline{\Omega}}^b = \tilde{k}^b \tag{43}$$

AS of the constrained households have to be higher than the stock of bubble and higher than the demand for credit by the unconstrained households, but equal to the sum of the two.

We have a system in $\tilde{x}$ and $\tilde{k}$ only which admits a unique solution

$$\tilde{k} = \{\frac{A-1}{A} \cdot \epsilon \cdot (1-\alpha) + \frac{\gamma}{A} \cdot \alpha \cdot (1-\lambda) + \lambda \cdot \alpha\}^{\frac{1}{1-\alpha}} \tag{44}$$
$$\tilde{x} = \{\frac{A-1}{A} \cdot \frac{1-\alpha}{1-\frac{\gamma}{A}} - \lambda \cdot \alpha\} \cdot \tilde{k}^{\alpha} \tag{45}$$

We need to check that the bubble is indeed feasible, in the sense that it does not exceed the savings of the constrained households, but that it is strictly positive

$$\frac{A-1}{A} \cdot \tilde{\underline{\Omega}} - \tilde{x} = \lambda \cdot \alpha \tag{46}$$
$$\tilde{x} > 0 \Leftrightarrow (1-\epsilon) > \frac{A-\gamma}{A-1} \cdot \frac{\alpha}{1-\alpha} \cdot \lambda \tag{47}$$

The condition for the bubbless equilibrium to be dynamically efficient is

$$\frac{A-\gamma}{A-1} \cdot \frac{\alpha}{1-\alpha} > 1 \tag{48}$$

Such that if λ is small enough and ϵ high enough, a bubble may appear in a otherwrise dynamically efficient economy provided that the steady-state is unequal. Furthermore we can check that as long as (46) holds, $\tilde{k}^b < \tilde{k}$ and thus $\tilde{R}^b > \tilde{R} > \tilde{\rho}^b = \tilde{q} = 1$. ■

$\tilde{x}$ is the maximal initial bubble which is compatible with a bubble in a otherwise dynamically efficient economy: if the initial bubble is higher $x_0 > \tilde{x}$, the interest rate on the credit market will be higher than one and the bubble would grow indefinitely. If the initial bubble is lower $x_0 < \tilde{x}$, the interest rate on the credit market will be lower than one and the stock bubble will asymptotically decrease to zero: the economy converges to the bubbless equilibrium we already found. So, bubbles in our model still crowd out investment as in Tirole (1985) because they reduce the amount of savings available for capital accumulation, but bubbles may appear in a otherwise dynamically efficient economy as long as the bubble is small enough. This is a new result which may seem somewhat surprising, but is due here to the co-existence of inequality in wealth among agents: inequality generate a spread between the rates of return on capital accumulation and on loans on the credit market, thus for the poors who are constrained to lend it may be profitable to buy the bubble even if it is not for the rich. Note that bubbles are associated to unequality in wealth among households, thus we can not concluce as in Tirole (1985) that bubbles are Pareto-improving.

5 Conclusion

We studied in the first part of this thesis the effects of inequality on capital accumulation and growth in the presence of imperfect capital markets and some indivibilities. We showed that if and only if the behaviors of the agents are linear with respect to wealth and under log-linear preferences, inequality do not matter at all for capital accumulation and aggregate wealth. Furthermore the steady-state may or may not be caracterized by an unequal distribution of wealth, depending on the degree of imperfection on the credit market and on the initial distribution of wealth.

Naturally some improvements could be done here: the log-linear preferences, through very tractable, clearly are unrealistic and may not capture the richness of the effects which could be induced by inequality. For example, with CRRA preferences, the consumption and savings behaviors would not only depend on wealth but also on the interest rate. As the two groups of agents face different interest rates and as those interest rates depend on the distribution of wealth, it could induce some aggregate effects of wealth inequality on the capital accumulation that are not observed with log-linear preferences.

The second part of this thesis dealt with financial bubbles and their possible interactions with growth and inequality. We showed that financial bubbles may appear under certain conditions in a otherwise dynamically efficient economy, which is a new result. Obviously in this case inequality may have some strong macroeconomic effect and redsitributive policies may favor financial stability. Furthermore we proved that bubbles market and credit market are sort of substitute to each other: if the bubbles are large enough, the credit market will not be necessary anymore, will nto exist.

An interesting improvement in the second part of the paper, which is also related to the first, would be to study occupational choice with different technology and imperfect but existant capital markets. We focused mostly on the effects of inequality on the capital accumulation, that is the level of investment in the economy, whereas another strong effect of inequalities seem to be on investment efficiency. It could be interesting to study if the results of Martin & Ventura (2012) still hold when one include a functioning but not perfect financial market.

To conclude our study we may say that we think we proved that inequality in the wealth distribution may favor the emergence in two distinct ways: first, inequality may promote over-capital accumulation and dynamic inefficiency; second, inequality may generate an interest rate spread between the capital and credit markets, which allows financial bubbles to exist even in otherwise dynamically efficient economies. Thus this thesis provides new arguments which may support fiscal policies which aim at redistributing the wealth.

A Appendix

Proof of proposition (9). When $\theta > 0$, the system of differential equations admits the following fixed point for $(\tilde{p}, \tilde{k})$:

$$\tilde{p} = \frac{B + C}{D + \frac{\gamma}{A} \cdot B + \frac{\gamma}{A} \cdot C\}} \tag{49}$$

$$\begin{aligned}
&\text{where} \quad B \equiv \lambda \cdot \alpha \cdot \{A + \frac{1-\alpha}{\alpha} \cdot \frac{1}{1-\lambda}\} \\
&\text{and} \quad C \equiv \theta \cdot (1-\alpha) \cdot \{(1-\epsilon) + \epsilon \cdot \frac{\lambda}{1-\lambda}\} \\
&\text{and} \quad D \equiv (1-\epsilon) \cdot (A-1) \cdot (1 - \frac{A-\gamma}{A} \cdot \theta) \cdot (1-\alpha)
\end{aligned}$$

$$\tilde{k} = \{\lambda \cdot \alpha \cdot \frac{D}{B} \cdot \frac{\epsilon}{(1-\epsilon)} + \frac{\gamma}{A} \cdot \alpha \cdot (1-\lambda) + \lambda \cdot \alpha \cdot \frac{1}{\tilde{p}}\}^{\frac{1}{1-\alpha}} \tag{50}$$

■

References

- Aghion & Bolton, "A Theory of Trickle-Down Growth and Development", *The Review of Economic Studies*, 1997
- Banerjee & Newman, "Occupational Choice and the Process of Development", *Journal of Political Economy*, 1993
- Banerjee, "Investment Efficiency and the Distribution of Wealth", *World Bank Commission*, 2007
- Blinder, "Distribution Effects and the Aggregate Consumption Function", *Journal of Political Economy*, 1975
- Cabarello, Fahri & Hammour, "Speculative Growth: Hints from the US Economy", *Working Paper*, 2005
- Caballero, Fahri & Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates", *American Economic Review*, 2008
- Carroll & Kimball, "On the Concavity of the Consumption Function", *Econometrica*, 1991
- Carrol & Kimball, "Precautionary Saving and Precautionary Wealth"; *The New Palgrave Dictionary of Economics, 2nd Ed.*, 2006
- Cahuc & Challe, "Produce or speculate? Asset bubbles, occupational choice and efficiency",*Working Paper*, 2009
- Chatterjee, "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model", *Journal of Public Economics*, 1994
- Fahri & Tirole, "Bubbly Liquidity", *Working Paper*, 2009
- Gathak & Morelli, "Occupational Choices and Dynamic Incentives", *The Review of Economic Studies*, 2001
- Jappelli & Pagano, "Savings, Growth and Liquidity Constraints", *Quaterly Journal of Economics*, 1994
- Martin & Ventura, "Economic Growth with Bubbles", *American Economic Review*, 2012
- Matsuyama, "Endogeneous Inequality", *The Review of Economic Studies*, 2000
- Michel, Lambrecht & Thibault, "Capital Accumulation and Fiscal Policy in an OLG Model with Family Altruism", *Journal of Public Economic Theory*, 2006
- Mino, "Growth and Bubbles with Consumption Externalities", *Working Paper*, 2007
- Olivier, "Growth-enhancing Bubbles", *International Economic Review*, 2000
- Piketty, "The Dynamic of the Wealth and the Interest Rate under Credit Rationning", *The Review of Economic Studies*, 1997
- Ray & Mookherjee, "Persistent Inequality", *The Review of Economic Studies*, 2003
- Samuelson, "An Exact Consumption-Loan Model of Interest Rates With or Without the Social Contrivance of Money", *Journal of Political Economy*, 1958

- Tirole, "Asset Bubbles and Overlapping Generations",*Econometrica*, 1985
- Zhou, "Rational Bubbles and The Spirit of Capitalism", *MPRA Working Papers*, 2011